On complemented copies of $c_0(\omega_1)$ in $C(K \times K)$ spaces

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Given a compact Hausdorff space K the geometry of the Banach space $C(K\times K)$, besides being interesting by itself, is additionally important because $C(K\times K)$ is isomorphic to the injective tensor product $C(K)\hat{\otimes}_\varepsilon C(K)$ and hence relevant to the investigations of the properties of these tensor products $X\hat{\otimes}_\varepsilon Y$ in terms of the properties of X and Y. It is well known, by a surprising and celebrated result of P. Cembranos and F. Freniche from 1984, that if $C(K)$ contains a copy of c_0 (i.e., C(K) is infinite dimensional), then $C(K\times K)$ always contains a complemented copy of c_0.

A nonseparable version of this result has been recently obtained by E. M. Galego and J. Hagler who proved that it is relatively consistent with ZFC that if $C(K)$ has density ω_1 and $C(K)$ has a copy of $c_0(\omega_1)$, then $C(K \times K)$ has a complemented copy $c_0(\omega_1)$. Their proof relies on Todorcevic's analysis of nonseparable Banach spaces under the assumption of an additional set-theoretic axiom known as Martin's Maximum.

In this paper, we show that this nonseparable version of Cembranos' and Freniche's theorem indeed is sensitive to additional set-theoretic assumptions. We prove that under the presence of the Ostaszewski's combinatorial principle $\clubsuit$, there is a scattered compact space K such that $C(K)$ has density ω_1, $C(K)$ contains a copy of $c_0(\omega_1)$, however, $C(K \times K)$ contains no complemented copy of $c_0(\omega_1)$. We also show show that for most known until now constructions of scattered compacta K similar to the one we consider, that is, with a copy of $c_0(\omega_1)$ but without any complemented copy, the space $C(K \times K)$ contains a complemented copy of $c_0(\omega_1)$.