On Nikodym-Grothendieck boundedness theorem.

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The talk is based on the joint work with Santiago Moll

Session: 36. Topology in Functional Analysis

In a recent paper, M. Valdivia has shown that if Ω is a compact k-dimensional interval in $\mathbb{R}^k$, $\mathcal{A}$ is the algebra of Jordan measurable subsets of Ω, $\mathcal{A} = \cup_n \mathcal{A}_n$, with $\mathcal{A}_n \subset \mathcal{A}_{n+1}$, $n \in \mathbb{N}$, then there exists $m \in \mathbb{N}$ such that if H is a set of bounded additive complex measures defined in $\mathcal{A}$ such that, for each $A \in \mathcal{A}_m$, $\sup\{|\lambda(A)| : \lambda \in H\} < \infty$, then $\sup\{|\lambda|(\Omega) : \lambda \in H\} < \infty$, where $|\lambda|$ is the variation of λ. M. Valdivia says that "The proof of this theorem can be extended to more general situations".

We will discuss some extension of this results. In particular we will prove that if $\mathcal{A}$ is a σ-algebra defined on a set Ω and if

- $\mathcal{A} = \bigcup_{n_1} \mathcal{A}_{n_1}$, with $\mathcal{A}_{n_1} \subset \mathcal{A}_{n_1+1}$, $n_1 \in \mathbb{N}$,
- each $\mathcal{A}_{n_1} = \cup_{n_2} \mathcal{A}_{n_1,n_2}$, with $\mathcal{A}_{n_1,n_2} \subset \mathcal{A}_{n_1,n_2+1}$, $(n_1, n_2) \in \mathbb{N}^2$,
- ....,
- and each $\mathcal{A}_{n_1,n_2,\cdots,n_{p-1}} = \bigcup_{n_p} \mathcal{A}_{n_1,n_2,\cdots,n_{p-1}n_p}$, with $\mathcal{A}_{n_1,n_2,\cdots,n_{p-1}n_p} \subset \mathcal{A}_{n_1,n_2,\cdots,n_{p-1}n_p+1}$, $(n_1, n_2, \cdots, n_p) \in \mathbb{N}^p$,

then there exists $(m_1, m_2, \cdots, m_p) \in \mathbb{N}^p$ such that if H is a set of bounded additive measures defined in $\mathcal{A}$ such that for each $A \in \mathcal{A}_{m_1,m_2,\cdots,m_{p-1}m_p}$, $\sup\{|\lambda(A)| : \lambda \in H\} < \infty$, then $\sup\{|\lambda|(\Omega) : \lambda \in H\} < \infty$, where $|\lambda|$ is the variation of λ. Additionally, if we continue this decomposition process then there exists a sequence $(m_q)_q$ such for each $p \in \mathbb{N}$ the finite sequence $(m_1, m_2, \cdots, m_p)$ has the previous boundedness property.

Some applications of this boundedness result will be presented.