On compact subspaces of the space of separately continuous functions with the cross-uniform topology

Oleksandr V. Maslyuchenko

Jurij Fedkovych Chernivtsi National University, Ukraine
ovmasl@gmail.com

Session: 36. Topology in Functional Analysis

In [1] the authors proposed a natural topologization of the space S of all separately continuous functions $f : [0,1]^2 \to \mathbb{R}$ called *the topology of the sectionally uniform convergence*. This topology can be considered on the space $S(X \times Y)$ of all separately continuous functions $f : X \times Y \to \mathbb{R}$ for any topological spaces X and Y. A base of this topology is given by the sets $W_{E,\varepsilon}(f_0) = \{f \in S(X \times Y) : \forall p \in \mathrm{cr}E \quad |f(p) - f_0(p)| < \varepsilon\}$, where E is a finite subset of $X \times Y$, $\varepsilon > 0$, $f_0 \in S(X \times Y)$ and $\mathrm{cr}E = (X \times \mathrm{pr}_Y(E)) \cup (\mathrm{pr}_X(E) \times Y)$ is *the cross* of the set E. We call this topology *the cross-uniform topology* and always endow the space $S(X \times Y)$ by this topology. If X and Y are compacta then $S(X \times Y)$ is a topological vector space. In [1] it was proved only that $S = S([0,1]^2)$ is a separable non-metrizable complete topological vector space, and the authors asked about the other properties of S. The following result was obtained in collaboration with the authors of [1].

Theorem 1. *Let X, Y be compacta without isolated points. Then $S(X \times Y)$ is meager and barreled.*

Another intrigued question on the space $S(X \times Y)$ is the problem on description of compact subspaces of $S(X \times Y)$ for any compacta X and Y. Compact subspaces of $B_1(X)$ (= the space of all Baire one function with the pointwise topology) are, so-called, Rosenthal compacta if X is a Polish space. Since $S([0,1]^2) \subseteq B_1([0,1]^2)$, we expected the appearance of some Rosenthal type compacta. But it turns out that the structure of compact subspaces of $S(X \times Y)$ is simpler. Let $w(X)$ denote the weight of a topological space X and let $c(X)$ denote the cellularity of X.

Theorem 2. *Let X, Y be infinity compacta and K be a compact. Then K embeds into $S(X \times Y)$ if and only if $w(K) \leq \min\{c(X), c(Y)\}$.*

References

[1] Voloshyn H.A., Maslyuchenko V.K. *The topologization of the space of separately continuous functions*, Carpathian Mathematical Publications 2013, 5(2), 199-207. (in Ukrainian)