On uncomplemented isometric copies of c_0 in spaces of continuous functions on products of the two-arrows space

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Phillips in [4] proved that c_0 is an uncomplemented subspace of l_∞. We do not find in the literature many classes of separable Hausdorff compact spaces K such that there exists a subspace X isomorphic to c_0 and uncomplemented in $C(K)$. Except $\beta\mathbb{N}$ appears essentially only the class of Mrówka spaces (see [3]). The reason is simple, usually it is quite hard to show the uncomplementability. There is one general method to do it, it is a modification of the Whitley proof of the Phillips theorem (see [6]). The method based on the facts that any $C(K)$ space, when K is separable and compact, does not contain any isomorphic copy of $c_0(\Gamma)$ for any uncountable set Γ but the quotient space $C(K)/X$ contains such a copy (see [2], [6], [1]).

We construct for every $n \geqslant 2$ a subspace X_n isometric to c_0 and complemented in $C(\mathbb{L}^n)$, the n-fold product of two arrows space $\mathbb{L}$, such that $\inf\{\|P\| : P : C(\mathbb{L}^n) \to X_n \text{ is a projection}\} \geqslant n+2$ and the quotient space $C(\mathbb{L}^n)/X_n$ has a $(3+4\sqrt{2})$ norming sequence of norm one functionals. The inequality together with the last fact enables us to find an isometric to c_0 and uncomplemented subspace Y of $C(\mathbb{L}^{\mathbb{N}})$ such that the quotient space $C(\mathbb{L}^{\mathbb{N}})/Y$ is isomorphic to a subspace of l_∞.

References

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