On a Gulisashvili question on scalarly measurable functions

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Let X be a Banach space, X^* be its dual and (Ω, Σ) be a measurable space. A function $\varphi : \Omega \to X$ is *scalarly measurable* if $f \circ \varphi$ is measurable for every $f \in X^*$. The φ is *totally scalarly measurable* if the set

$$F = \{f \in X^* : f \circ \varphi \text{ is measurable}\}$$

is total, i.e. for every $x \in X,\ x \neq 0$, there exists $f \in F$ such that $f(x) \neq 0$. A Banach space X satisfies the *property* $\mathcal{D}$ if for every measurable space (Ω, Σ) every totally scalarly measurable function $\varphi : \Omega \to X$ is, in fact, scalarly measurable.

Property $\mathcal{D}$ has been introduced by A. Gulisashvili [1] in connection with the Pettis integral in interpolation spaces. Gulisashvili has proved that the weak* angelicity of X^* implies the property $\mathcal{D}$ of X and has risen the problem of the reverse implication. We answer the Gulisashvili problem in negative.

References

[1] A. Gulisashvili, *Estimates for the Pettis integral in interpolation spaces and inversion of the embedding theorems*, Dokl. Acad. Nauk SSSR 263, 1982, 793–798 (Russian). English transl.: Sovet. Math. Dokl. 25, 1982, 428–432.