The obstruction to contractibility of Snake cones and Alternating cones

Andreas Zastrow

Institute of Mathematics, University of Gdansk, Poland
zastrow@mat.ug.edu.pl

The talk is based on joint work with K. Eda, U. Karimov and D. Repovš

Session: 37. Wild algebraic and geometric topology

By Whitehead's Theorem it is clear that for CW-complexes any obstruction to contractibility will be in the homotopy groups. For general spaces this is naturally wrong. Also for Peano-continua there exists an infinite-dimensional counterexample by [2]. The question, whether also a finite-dimensional counterexample is possible, is still unanswered. The talk will discuss the construction techniques of some spaces which are somehow closest to providing corresponding counterexamples. The four construction principles (Abbr.: *SC, CSC, AC, CAC)* to be discussed turn a given space X into a space of one dimension higher in a way that the fundamental group becomes trivial, but the obstruction to contractibility may be expected to be retained. If applied to a one-dimensional manifold X a two-dimensional wild complex arises that has no closed surface as a subcomplex. However, in spite of this, π_2 will be non-trivial. The talk (mainly presenting the results of [1], but also repeating the results of some predecessor-papers without the speaker's participation) will introduce and compare these functors *SC*, *CSC*, *AC* and *CAC*, explain why in general the results are not homotopy equivalent, but in spite of this $SC(S^1) \simeq CSC(S^1) \simeq AC(S^1) \simeq CAC(S^1)$. In order to obtain this result it was necessary to compute the second homology group of a Hawaiian Earring-type product of the torus-surface.

References

[1] K. Eda., U. H. Karimov, D. Repovš, A. Zastrow, *On snake cones, alternating cones and related constructions*, Glas. Mat. Ser. III 48(68) (2013), no. 1, 115–135.

[2] U. H. Karimov, D. Repovš, *On noncontractible compacta with trivial homology and homotopy groups*, Proc. Amer. Math. Soc. 138 (2010), no. 4, 1525–1531.