Three critical point theorems with applications to nonlinear BVPs

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In this talk we are concerned with three critical theorems applicable for C^1 action functionals connected to anisotropic problems. Results are based on recent investigations and on ideas developed by Ricerri which can be summarized as follows: Let $(X, \|.\|)$ be a uniformly convex Banach space with strictly convex dual, $J \in C^1(X)$ be a functional with compact derivative, $x_0, x_1 \in X$, $p, r \in \mathbb{R}$, $p > 1$, $r > 0$. Assume

(A.1) $\liminf\limits_{\|x\|\to\infty} \frac{J(x)}{\|x\|^p} \geq 0$;

(A.2) $\inf\limits_{x\in X} J(x) < \inf\limits_{\|x-x_0\|\leq r} J(x)$;

(A.3) $\|x_1 - x_0\| < r$ and $J(x_1) < \inf\limits_{\|x-x_0\|=r} J(x)$.

There exists a nonempty open set $A \subseteq (0, +\infty)$ s. t. for all $\lambda \in A$ the functional $x \to \dfrac{\|x - x_0\|^p}{p} + \lambda J(x)$ has at least three critical points in X.

Main idea used in this talk are concerned with the following

- replace the term $\|x\|^p$ with some convex coercive functional
- obtain a more precise estimation on the set A
- examine applicability of new results
- generalize to the locally Lipschitz case
- check what happens when the space is finite dimensional