Conley index of invariant sets for strongly damped hyperbolic equations at resonance

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We study the existence of compact invariant sets for the strongly damped hyperbolic differential equation $\ddot{u}(t) = -Au(t) - cA\dot{u}(t) + \lambda u(t) + F(u(t))$ being at resonance at infinity, that is, $A : X \supset D(A) \to X$ is a sectorial operator on a Banach space X and $F : X^\alpha \to X$ is a continuous bounded map defined on the fractional space X^α associated with A, $c > 0$ is a damping factor and λ is an eigenvalue of A. We provide two geometrical assumptions for the nonlinearity F, that allow to obtain Conley index formulas stating that the Conley index for the associated semiflow, with respect to large ball, is equal to suspension of the sphere of proper dimension depending on which of the geometrical assumptions imposed on the nonlinearity is satisfied. It will be also proved that the geometrical assumptions generalize well-known Landesman-Lazer conditions (see e.g. [1], [2]), and moreover, cover some other cases where the nonlinearity F exhibits a lower order resonance at infinity (see e.g. [3], [6]). Presented topic is a continuation of [4] where the problem of existence of compact invariant sets is studied for nonlinear parabolic equations.

References

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