Minimal energy solutions for repulsive nonlinear Schrödinger systems

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We establish existence and nonexistence results concerning fully nontrivial minimal energy solutions of the nonlinear Schrödinger system

$$\begin{aligned} -\Delta u + \quad u &= |u|^{2q-2}u + b|u|^{q-2}u|v|^q \quad \text{in } \mathbb{R}^n, \\ -\Delta v + \omega^2 v &= |v|^{2q-2}v + b|u|^q|v|^{q-2}v \quad \text{in } \mathbb{R}^n. \end{aligned}$$

We consider the repulsive case $b < 0$ and assume that the exponent q satisfies $1 < q < \frac{n}{n-2}$ in case $n \geq 3$ and $1 < q < \infty$ in case $n = 1$ or $n = 2$. For space dimensions $n \geq 2$ and arbitrary $b < 0$ we prove the existence of fully nontrivial nonnegative solutions which converge to a solution of some optimal partition problem as $b \to -\infty$. In case $n = 1$ we prove that minimal energy solutions exist provided the coupling parameter b has small absolute value whereas fully nontrivial solutions do not exist if $1 < q \leq 2$ and b has large absolute value. This generalizes the existence results found in [1].

References

[1] B. Sirakov: Least energy solitary waves for a system of nonlinear Schrödinger equations in $\mathbb{R}^n$, Comm. Math. Phys. 271 (2007), 199–221.

[2] R. Mandel: Minimal energy solutions for repulsive nonlinear Schrödinger systems, http://arxiv.org/abs/1303.4521