Uniform bounds for strongly competing systems: the optimal Lipschitz case

Nicola Soave

Justus Liebig University Giessen, Germany
nicola.soave@gmail.com

This is a joint work with Alessandro Zilio (Politecnico di Milano).

Session: 38. Variational Methods in Nonlinear Analysis

We present uniform regularity results regarding positive solutions of the family of systems

$$\begin{cases} -\Delta u_{i,\beta} = f_{i,\beta}(u_{i,\beta}) - \beta u_{i,\beta} \sum_{j\neq i} a_{ij} u_{j,\beta}^p & \text{in } \Omega \\ u_{i,\beta} = 0 & \text{on } \partial\Omega \qquad i = 1, \dots, k \end{cases}$$

in the cases $p = 1$ (symmetric interaction) and $p = 2$ (variational interaction). For such systems, of interest in population dynamics and in the study of phase-separation of Bose-Einstein condensates, we show that $L^\infty(\Omega)$-boundedness implies $\mathcal{C}^{0,1}(\overline{\Omega})$-boundedness, uniformly in $\beta \to +\infty$. This extend the $\mathcal{C}^{0,\alpha}$-regularity theory available in the literature ($0 \leq \alpha < 1$) to the optimal Lipschitz case.