A dual variational approach to nonlinear Helmholtz equations

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We set up a dual variational framework to detect real standing wave solutions of the nonlinear Helmholtz equation

$$-\Delta u - k^2 u = Q(x)|u|^{p-2}u, \qquad u \in W^{2,p}(\mathbb{R}^N)$$

with $N \geq 3$, $\frac{2(N+1)}{(N-1)} < p < \frac{2N}{N-2}$ and nonnegative $Q \in L^\infty(\mathbb{R}^N)$. We prove the existence of nontrivial solutions for periodic Q as well as in the case where $Q(x) \to 0$ as $|x| \to \infty$. Classical direct methods in critical point theory do not apply to this problem due to the lack of Fredholm properties. In the periodic case, a key ingredient of the approach is a new nonvanishing theorem related to an associated integral equation. The solutions we study are superpositions of outgoing and incoming waves and are characterized by a nonlinear far field relation.