

Expanding maps and Anosov diffeomorphisms on infra-nilmanifolds

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Infra-nilmanifolds play an important role in dynamical systems, especially when studying expanding maps or Anosov diffeomorphisms. Because of the algebraic nature of these manifolds, questions about self-maps can be translated into questions about endomorphisms of their fundamental group. In this way, it was shown by M. Gromov in [3] that every expanding map on a closed manifold is topologically conjugate to an affine infra-nilendomorphism. Up till now it is unknown if a similar statement also holds for Anosov diffeomorphisms, although some partial results point in that direction, e.g. for infra-nilmanifolds and for codimension one Anosov diffeomorphisms.

These results motivate the study of infra-nilmanifolds admitting an Anosov diffeomorphism or an expanding map. In this talk I will show how these questions are related to the existence of certain automorphisms of nilpotent Lie algebras. As a consequence, we can classify all infra-nilmanifolds modeled on a free nilpotent Lie group admitting an Anosov diffeomorphism, see [2]. Another consequence is that for nilmanifolds, the existence of an expanding map or a non-trivial self-covering does only depend on the commensurability class of the fundamental group of the manifold, answering a question stated in [1]. This allows us to construct examples of nilmanifolds admitting an Anosov diffeomorphism but no expanding map.

References

- [1] I. Belegradek, *On co-Hopfian nilpotent groups*, The Bulletin of the London Mathematical Society 6, 2003, 805–811.
- [2] K. Dekimpe, J. Deré, *Existence of Anosov diffeomorphisms on infra-nilmanifolds modeled on free nilpotent Lie groups*, preprint, 2013, arXiv:1304.6529.
- [3] M. Gromov, *Groups of polynomial growth and expanding maps*, Institut des Hautes Études Scientifiques 53, 1981, 53–73.