Minimal projections in three-dimensional normed spaces

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By a result of Bohnenblust for every three-dimensional normed space X and its two-dimensional subspace Y, there exists a projection $P\colon X \to Y$ such that $\|P\| \leq \frac{4}{3}$. The aim of the talk is to give a sketch of the proof of the following theorem: if for some subspace Y the minimal projection $P\colon X \to Y$ satisfies $\|P\| \geq \frac{4}{3} - R$ for some $R > 0$, then there exists two dimensional subspace Z of X and projection $Q\colon X \to Z$ for which $\|Q\| \leq 1 + \Phi(R)$ where $\Phi(R) \to 0$ as $R \to 0$. In other words, every space which has a subspace of almost maximal projection constant has also a subspace of almost minimal projection constant. As a consequence, every three-dimensional space has a subspace with the projection constant strictly less than $\frac{4}{3}$, which gives a non-trivial upper bound for the problem posed by Bosznay and Garay. We shall also characterize all three-dimensional spaces which have a subspace with the projection constant equal to $\frac{4}{3}$.