On the nondegenerate jumps of the Lojasiewicz exponent

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Give the session name!

Let $f_0 \colon (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ be an isolated singularity. We define the number

$$\mathcal{L}_0(f) := \inf\{\alpha \in \mathbb{R}_+ : \exists_{C>0} \exists_{r>0} \forall_{\|z\|<r} \|\nabla f_0(z)\| \geq C\|z\|^\alpha\}$$

and call it the Lojasiewicz exponent of f_0. In [1], B. Teissier calculated $\mathcal{L}_0(f_0)$ in terms of polar invariants of the singularity f_0 and proved that $\mathcal{L}_0(f_0)$ is lower semicontinuous in any μ-constant deformation of the singularity f_0. A. Płoski generalized his result and proved that the Lojasiewicz exponent is lower semicontinuous in any multiplicity-constant deformation of a finite holomorphic map germ (see [2]). B. Teissier also showed that if we do not assume μ-constancy, then $\mathcal{L}_0(f_0)$ is neither upper or lower semicontinuous (see [3]). The "jump phenomena" of the Lojasiewicz exponent were rediscovered by some authors (see [4]). The aim of this talk is to give formulas for jump upwards and downwards of $\mathcal{L}_0(f_0)$ in nondegenerate class of curves singularities in terms of the Newton diagram of f_0. By the jump downards of $\mathcal{L}_0(f_0)$ we mean the minimum non-zero positive difference between the Łojasiewicz exponent of f_0 and one of its deformations (f_s). We define in analogous way the jump upwards. We also indicate the deformations, in which the jumps are attained.

References

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[4] J. Mc Neal, A. Némethi, *The order of contact of a holomorphic ideal in* $\mathbb{C}^2$, Math. Z. 250, 2005, 873–883.